Normalized gradient-based inverse compositional Gauss-Newton (NG-IC-GN) algorithm for digital image correlation under non-uniform illumination variations

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1. INTRODUCTION

The introduction of digital image correlation (DIC) techniques has opened the way for studying full-field kinematics (either displacement or strain) of complex structures using low-cost optical measurements [1]. By choosing a subset in the reference image, DIC searches for the most similar subset in the deformed image according to certain correlation criterion [2]. These techniques result in a non-linear criterion to be minimized with respect to the parameters (local displacement or homogenous strain over the subset). The minimization is usually performed by a Newton-Raphson algorithm. One inherent limitation of the Newton-Raphson method is that the Hessian matrix and its inverse are updated in each iteration. Baker and Matthews proposed the efficient IC-GN algorithm in the field of compute vision [3]. The most interesting property of the IC-GN algorithm is that the Hessian matrix needs only to be calculated once during the iterations for each subset. Pan et al. applied the IC-GN to optimize a ZNSSD criterion and obtained remarkable results for DIC application [4] in the case of scale and offset of lights. The real environment light is complex and non-uniform. We propose a different DIC algorithm, the normalized gradient-based inverse composition Gauss-Newton (NG-IC-GN) algorithm, to overcome a non-uniform change of the image intensities by minimizing the SSD of gradients criterion. The NG-IC-GN is insensitive to the image intensity changes due to the insensitivity of the image gradient to the illumination variation. The present algorithm inherits the advantages of the original IC-GN and optimizes the simple SSD, instead of ZNSSD criterion, to obtain the deformation parameters.

2. The formulation of the NG-IC-GN

The proposed NG-IC-GN algorithm is in the framework of the IC-GN algorithm [3]. The main difference between these two algorithms is that the normalized gradient is used in the NG-IC-GN while the intensity is used in the IC-GN. For brevity, we only introduce the NG-IC-GN algorithm. For more details about the IC-GN algorithm, we refer to [4,5]. First of all, we define the normalized gradients of an image as follows:

$$\begin{cases} T_x(x_i, y_i) = g_{xi} / (g_i + g_m) \\ T_y(x_i, y_i) = g_{xi} / (g_i + g_m) \end{cases} \tag{1}$$

with $g_i = \sqrt{(g_{xi}^2 + g_{yi}^2)}$ and $g_m = \sum_{i=1}^{n} g_i / n$. g_{xi} and g_{yi} are the gradients of the intensity field at point (x_i, y_i).

The normalized criterion is given below:

$$C_{NGSSD} = \sum_{i=1}^{N} \left\{ \left[Fx\left(\mathbf{x} + \mathbf{w}(\xi; \Delta\mathbf{P})\right) - Gx\left(\mathbf{x} + \mathbf{w}(\xi; \mathbf{P})\right) \right]^2 + \left[Fy\left(\mathbf{x} + \mathbf{w}(\xi; \Delta\mathbf{P})\right) - Gy\left(\mathbf{x} + \mathbf{w}(\xi; \mathbf{P})\right) \right]^2 \right\} \tag{2}$$

where $\xi = (\Delta x, \Delta y, 1)^T$ is the local coordinates of the pixel points in each subset. The vector $\mathbf{P} = \left(u, u_x, u_y, v, v_x, v_y\right)^T$ is deformed parameter. The incremental deformed parameter $\Delta\mathbf{P} = \left(\Delta u, \Delta u_x, \Delta u_y, \Delta v, \Delta v_x, \Delta v_y\right)^T$.

After taking a Taylor expansion and minimization of Eq. (2), we obtain the incremental parameter:

$$\begin{aligned} \Delta\mathbf{P} = -\mathbf{H}^{-1} \sum_{i=1}^{N} \Bigg\{ & \left(\nabla Fx \left(\frac{\partial \mathbf{w}}{\partial \mathbf{P}} \right)^T \right) \left[Fx(\mathbf{x} + \xi) - Gx\left(\mathbf{x} + \mathbf{w}(\xi; \mathbf{P})\right) \right]^2 \\ & + \left(\nabla Fy \left(\frac{\partial \mathbf{w}}{\partial \mathbf{P}} \right)^T \right) \left[Fy(\mathbf{x} + \xi) - Gy\left(\mathbf{x} + \mathbf{w}(\xi; \mathbf{P})\right) \right]^2 \Bigg\} \end{aligned} \tag{3}$$

$$\mathbf{H} = \sum_{i=1}^{N} \left\{ \left(\nabla Fx \left(\frac{\partial \mathbf{w}}{\partial \mathbf{P}} \right)^{T} \right) \nabla Fx \left(\frac{\partial \mathbf{w}}{\partial \mathbf{P}} \right) + \left(\nabla Fy \left(\frac{\partial \mathbf{w}}{\partial \mathbf{P}} \right)^{T} \right) \nabla Fy \left(\frac{\partial \mathbf{w}}{\partial \mathbf{P}} \right)^{T} \right\} \tag{4}$$

So the Hessian matrix **H** only need to be calculated once before the iterations since ∇Fx and ∇Fy are dependent of the reference images. The updating warp function to update the target subset is written as

$$\mathbf{w}(\xi;\mathbf{P}_{new}) = \mathbf{w}(\xi;\mathbf{P}_{old})\left(\mathbf{w}(\xi;\Delta\mathbf{P})\right)^{-1} \tag{5}$$

3. Numerical example

We conduct a numerical analysis of strained speckle images with the sizes of 256×256 pixels.

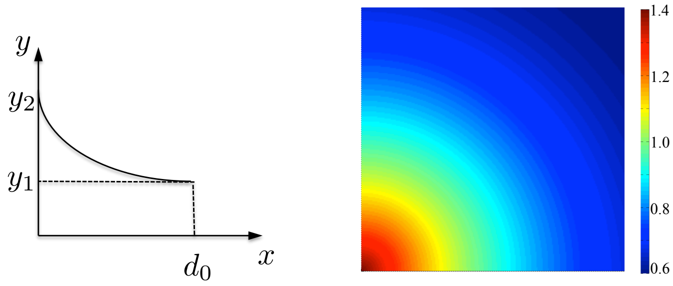

Figure 1. Non-linear space-based illumination function and corresponding contour

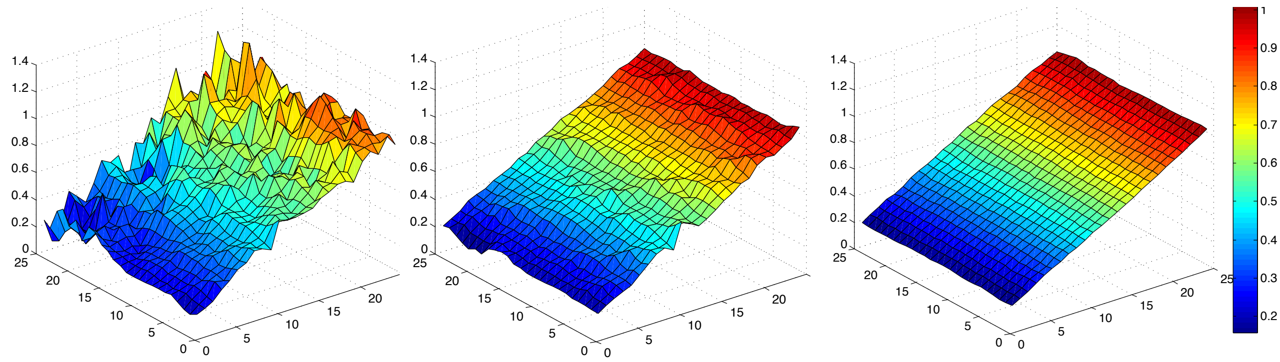

Figure 2. U-displacement. Left: IC-GN with SSD; Middle: IC-GN with ZNSSD; Right: The proposed method

3. CONCLUSION

In the present work, we propose a novel NG-IC-GN algorithm for DIC-based kinematics measurements. This algorithm copes well with the DIC images with non-linear illumination intensity variation. First, the proposed algorithm has no limit that the denominator should not be close to zero which is demanded in the ZNSSD criterion. Second, the proposed algorithm produces much better results than the intensity-based IC-GN algorithm under non-linear intensity change.

References

[1] M.A. Sutton, W.J. Walters, W.H. Peters, W.F. Ranson, S.R. Mcneil, Determination of displacements using an improved digital correlation method, Image and Vision Computing, 1, pp.133–139, 1983.

[2] M.A. Sutton, J.J. Orteu, H.W. Schreier, *Image Correlation for Shape, Motion and Deformation Measurements*, Springer New-York, 2009. ISBN 978-0-387-78746-6.

[3] S. Baker, I. Matthews, Lucas-Kanada 20 years on: a unifying framework, International Journal of Computer Vision, 56, pp. 221-225, 2004.

[4] B. Pan, K. Li, W. Tong, Fast, robust and accurate digital image correlation calculation without redundant computations, Experimental Mechanics, 53, pp. 1277-1289, 2013.